2020/TDC (CBCS)/ODD/SEM/
MTMSEC–301T (A/B/C)/333A

TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

MATHEMATICS

(3rd Semester)

Course No. : MTMSEC–301T

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Honours students will answer *either* Group—A *or* Group—B and Pass students will answer Group—C

GROUP—A

(For Honours Students)

Course No. : MTMSEC–301T (H)

(Logic and Sets)

SECTION—A

Answer any *fifteen* of the following questions : 1×15=15

1. Classify each of the following statements as True or False :

(i) 8 5 or 2 1

(ii) 5 is prime 15 is prime

2. Write the negation of the following statement :

x $\mathbb{N}$ and $2x$ 5 0

3. Write the contrapositive of the following :

If n is odd integer, then n^2 n 2 is an even integer.

4. Write the converse of the following :

If x $\mathbb{R}$, then x^2 1 0.

5. If p is false, q is true, then what is the truth value of p $\sim q$?

6. Rewrite the following statement as an implication :

The diagonals of a parallelogram bisect each other.

7. Write the negation of the following :

For every real number x, there is an integer n such that n x.

8. Rewrite using quantifiers :

Every positive integer greater than 1 is a product of primes.

9. If o denotes contradiction, then show that p o is logically equivalent to p.

10. Using quantifiers, describe the case when a set A is not bounded below in $\mathbb{R}$.

11. Show that p $(\sim(p$ $q))$ is a tautology.

12. Construct the truth table for p $\sim q$.

13. Let A $\{1, 100, 200, 300, 400\}$. How many subsets of A have three elements?

14. What are the elements of $\mathbb{N}$ $(-5, 5)$?

15. Justify True or False :

$A \subsetneq B$, B C $A \subsetneq C$

16. If A , then write $P(P(A))$ in roster form.

17. Let A_n $\{a$ $\mathbb{Z} \mid a$ $n\}$. If A_3 A_5 A_k, then what is the value of k?

18. How many 3-digit numbers can be formed with the digits 1, 3, 5, 7?

19. Find the symmetric difference of the sets A $\{1, 2, 3, 4\}$ and B $\{2, 4, 6, 8\}$.

20. If A $\{x$ $\mathbb{Z} \mid 2x^3$ $9x^2$ $5x$ $0\}$, list the elements of $P(A)$.

21. Let A_n $\{1, 2, \cdots, n\}$ n $\mathbb{N}$. What is $\bigcup_{n\ 1} A_n$?

22. Fill in the blank :

A B A iff ____.

23. Justify True or False :

$A \setminus B$ A

24. If (a, b) and (c, d) are intervals with non-empty intersection, then (a, b) (c, d) (x, y). What are the values of x and y?

25. Define reflexive relation on a set.

26. Write a partition of the set A $\{1, 2, 3, 4\}$.

27. Justify if the relation R on $\mathbb{Z}$ defined by (a, b) R iff a b 0 is an equivalence relation.

28. Let X $\{a, b, c, d\}$. Write a relation on X that is symmetric but neither transitive nor reflexive.

29. Define poset.

30. Define maximal element of a poset.

SECTION—B

Answer any *five* of the following questions : 2×5=10

31. Show that $q \quad (p \quad q)$ is a tautology.

32. Construct the truth table for $p \quad (\sim q \quad p)$.

33. Using algebra of propositions, show that

$$(\sim (p \quad q)) \quad (\sim p \quad q)$$

is logically equivalent to $\sim p$.

34. A sequence $\langle x_n \rangle$ converges to x in $\mathbb{R}$ if for every 0, $n_0 \quad \mathbb{N}$ such that

$$n \quad n_0 \quad |x_n \quad x|$$

Use quantifiers to describe the case when $\langle x_n \rangle$ does not converge to x.

35. How many numbers from 1 to 1000 are neither divisible by 2 nor divisible by 5?

36. Show that for any three sets A, B and C

$$A \quad (B \quad C) \quad (A \quad B) \quad (A \quad C)$$

37. Prove or disprove :

$$A \quad B \qquad P(A) \quad P(B)$$

38. Let $A_n \quad \{x \quad \mathbb{Z} \mid x \quad n\}$ for each $n \quad \mathbb{N}$.

What is $\bigcap_{n \quad 1} A_n$?

39. Consider the equivalence relation on $\mathbb{Z} \setminus \{0\}$ defined by $a \sim b$ iff $ab \quad 0$. Determine the partition of $\mathbb{Z}$ corresponding to $\sim$.

40. Draw the Hasse diagram for the partial order on $\{0, 1, 2, 3\}$.

SECTION—C

Answer *any* **five** questions

41. Using truth tables, show that

$$((p \quad q) \quad (q \quad r)) \quad (p \quad r)$$

is a tautology. 5

42. Suppose that a is a non-zero rational number and that b is an irrational number. Prove that ab is irrational. 5

43. Using algebra of propositions, prove that

$$(s \quad ((\sim p \quad q) \quad r))$$

is logically equivalent to $\sim ((p \quad (\sim (q \quad r))) \quad s)$. 5

44. Using truth table, establish the following logical equivalences : 2½+2½=5

(i) $p \quad (p \quad q) \quad p$

(ii) $p \quad (p \quad q) \quad p$

45. Show that the union of two finite sets is finite. 5

46. Construct a bijection from (1, 2) to (21, 2021). Justify your answer. 5

47. For any sets A, B, C, show that—

(i) $(A \quad B) \quad C \quad (A \quad C) \quad (B \quad C)$

(ii) $A \setminus (B \setminus C) \quad (A \setminus B) \quad (A \setminus C^c)$ 3+2=5

48. *(a)* Show that

$$\bigcap_{n \quad 1} 1, 1 \quad \frac{1}{n} \quad \{1\}$$ 3

(b) Write $\mathbb{R} \setminus \mathbb{N}$ as a union of open intervals. 2

49. State and prove the fundamental theorem of equivalence relations. 5

50. Determine whether each partial order below is a total order : 2½+2½=5

(i) $(\mathbb{N} \quad \mathbb{N}, \quad)$ where $(a, b) \quad (c, d)$ iff $a \quad c$

(ii) $(\mathbb{N} \quad \mathbb{N}, \quad)$ where $(a, b) \quad (c, d)$ iff $a \quad c$ and $b \quad d$

GROUP—B

(For Honours Students)

Course No. : MTMSEC–301T (H)

(Programming in C)

SECTION—A

Answer any *fifteen* of the following questions : 1×15=15

1. What is C character set?

2. What are C tokens?

3. Write the general syntax of declaring variables in C.

4. What is a relational expression?

5. What are the high priority operators in C?

6. Write the general form of printf statement.

7. What are the three logical operators in C?

8. If initial value of x is 7, what will be its value after executing the statement $x \quad 3$; ?

9. What is the value on evaluating the expression 25 / 3 % 2?

10. Write the following as a C expression :

b^2 4*ac* is greater than zero.

11. Determine the value of the logical expression

a *b* && *a* *c*

if *a* 5, *b* 10, *c* 6.

12. Identify the error in the statement

printf (x = % d, x);

13. Write the general syntax of if-else statement.

14. What is the purpose of the break statement?

15. Write the general syntax of while loop.

16. What is the general syntax of the for loop?

17. What will happen when the following loop is executed?

```
while (5   3)
  {
    printf ("Hello");
  }
```

18. If initial value of sum is 2, what will be its value after the execution of the following loop?

```
for (i = 1; i   3; i++)
      sum = sum + i;
```

19. What type of function is the "main" function?

20. What is the type specifier of a function that does not return any value?

21. Write the function prototype declaration of a function *F* of integer type that accepts two integer arguments.

22. What is a function which calls itself known as?

23. Write True or False :

The names of the formal and actual arguments of a function should be same.

24. Mention two library functions of C.

25. Define an array.

26. Write the declaration of an integer array of size 10.

27. What should be the type of an array index?

28. A two-dimensional array is declared as

int x[3] [2] = {{1, 2}, {3, 4}, {5, 6}}

What are the values of x[0] [1] and x[1] [1]?

29. Point out the error in the declaration

float y (3);

30. Write the general form of declaring a multidimensional array.

SECTION—B

Answer any *five* of the following questions : 2×5=10

31. Write the rules for naming identifiers in C.

32. List all the 8 categories of operators in C.

33. Write a program to compute the average of three numbers.

34. Explain the precedence of arithmetic operators in C.

35. Explain entry-controlled and exit-controlled loops.

36. Write a program to test whether a given number is odd or even.

37. What is a function prototype? What is its purpose?

38. Write a simple program to compute the sum of two numbers using a user-defined function.

39. Explain the process of declaration and initialization of a one-dimensional array.

40. Write a simple program to input an array and display it.

SECTION—C

Answer *any* **five** questions

41. *(a)* Describe all the primary data types in C. 3

(b) Describe how the increment and decrement operators differ from each other. 2

42. *(a)* Explain formatted input with scanf function. 2

(b) Explain associativity of operators in C. 3

43. *(a)* Write a C program to convert temperature from Centigrade to Fahrenheit. 2

(b) Describe the logical operators in C. 3

44. *(a)* Write a C program to compute the area and perimeter of a rectangle taking length and breadth as inputs. 3

(b) Describe the arithmetic operators in C. 2

45. Write a C program to find the roots of a given quadratic equation. 5

46. Write a program to compute the sum of first *n* natural numbers using a loop. 5

47. *(a)* Discuss the need and importance of user-defined functions. 3

(b) Explain actual arguments and formal arguments of a function. 2

48. Write a program to compute the factorial of a positive integer using recursion. 5

49. Write a program to find the sum of two arrays. 5

50. *(a)* How are two-dimensional arrays initialized? What happens if some values are missing in an initializer? 2

(b) Write a short note on one-dimensional arrays. 3

GROUP—C

(For Pass Students)

Course No. : MTMSEC–301T (P)

(Classical Algebra and Trigonometry)

SECTION—A

Answer any *fifteen* of the following as directed : 1×15=15

1. Define adjoint of a square matrix *A*.

2. Define idempotent matrix.

3. Give an example of symmetric matrix.

4. Define involutory matrix.

5. When is a square matrix *A* said to be orthogonal?

6. Define inverse of a square matrix.

7. What is the rank of zero matrix?

8. When is a matrix said to be in normal form?

9. What is the relation between rank of the original matrix and its transpose?

10. What is the rank of the following matrix?

$$\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix}$$

11. The rank of a matrix in echelon form is equal to the number of non-zero rows of the matrix.

(Write True or False)

12. The elementary transformation alters the rank of the matrix.

(Write True or False)

13. If , , are the roots of the equation

$$x^3 \quad px^2 \quad qx \quad 0$$

then what is the value of ?

14. Write down the equation whose roots are $1, -2, 3, -4$.

15. Write down the equation whose roots are the roots of the equation

$$x^7 \quad 3x^5 \quad x^3 \quad x^2 \quad 7x \quad 2 \quad 0$$

with their signs changed.

16. Write down the sum of the roots of the equation $x^3 \quad px^2 \quad qx \quad p \quad 0$.

17. If i be the roots of the equation

$$ax^2 \quad bx \quad c \quad 0$$

then what is the other root of the equation?

18. Find the equation whose roots are reciprocal of the equation $x^2 \quad 2x \quad 1 \quad 0$.

19. Write down the value of $(\cos \quad i\sin \ )^{\ n}$.

20. Express $1 \quad i$ in polar form.

21. Write down the expansion of $\sin x$.

22. Write down the expansion of $\cos n$.

23. Write down the exponential value of $\sin x$.

24. Find the value of $e^{2n\ i}$.

25. Write down the expansion of $\tan^{\ 1} x$.

26. What is the value of the following series?

$$1 \quad \frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{7} \quad \frac{1}{9} \quad \cdots$$

27. Define $\sin hx$.

28. Write down the relation between $\tan ix$ and $\tan hx$.

29. Write down the sum of

$$\sin \quad \sin(\quad) \cdots \sin(\quad \overline{n \quad 1})$$

30. Write down the sum of

$$\text{cosec} \quad \text{cosec } 2 \quad \text{cosec } 2^2 \quad \cdots \quad \text{cosec } 2^{n \quad 1}$$

SECTION—B

Answer any *five* of the following questions : 2×5=10

31. If A , then show that adj (adj A) = A.

32. If A and B are any two matrices, then prove that $AB^T \quad BA^T$ is skew-symmetric matrix.

33. Define echelon form of a matrix with example.

34. Find the rank of the matrix

$$\begin{matrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ 1 & 2 & 2 \end{matrix}$$

35. The sum of two roots of the equation

$$x^3 \quad a_1x^2 \quad a_2x \quad a_3 \quad 0$$

is zero. Show that $a_1a_2 \quad a_3 \quad 0$.

36. Form an equation whose roots are decreased by 2 than the roots of the equation

$$x^3 \quad 3x^2 \quad 2x \quad 5 \quad 0$$

37. If n be a positive integer, then prove that

$$(1 \quad i)^n \quad (1 \quad i)^n \quad 2^{\frac{n}{2} \quad 1} \cos\frac{n}{4}$$

38. Show that

$$\frac{2}{2 \quad 4} \quad \frac{4}{2 \quad 4 \quad 6 \quad 8} \quad \frac{6}{2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12} \quad \cdots \quad 1$$

39. Separate real and imaginary parts (x and y being real) of $\tan(x \quad iy)$.

40. Find the sum of the series

$$1 \quad \frac{1}{3 \quad 2^2} \quad \frac{1}{5 \quad 2^4} \quad \cdots$$

SECTION—C

Answer any *five* of the following questions : 5×5=25

41. State and prove Jacobi's theorem.

42. If A and B are two invertible square matrices of the same order, then prove that AB is invertible. Also prove that $(AB)^{\quad 1} \quad B^{\quad 1}A^{\quad 1}$.

43. Find the rank of

$$\begin{matrix} 2 & 2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & 1 & 0 & 3 \\ 1 & 2 & 1 & 2 \end{matrix}$$

by reducing it to normal form.

44. Solve the following system of linear equations by matrix method :

$$\begin{matrix} x & 2y & 3z & 11 \\ x & 2y & 3z & 3 \\ x & 2y & 3z & 1 \end{matrix}$$

45. If , , are the roots of the equation

$$x^3 \quad px^2 \quad qx \quad r \quad 0$$

then find the equation whose roots are 2 , 2 , 2 .

46. Find the conditions if the roots of the equation

$$x^3 \quad px^2 \quad qx \quad p \quad 0$$

are in GP.

47. If sin sin sin cos cos cos 0, then prove that

$$\begin{matrix} \cos 3 & \cos 3 & \cos 3 & 3\cos(&) \\ \sin 3 & \sin 3 & \sin 3 & 3\sin(&) \end{matrix}$$

48. If $x \quad \log\tan \frac{}{4} \quad \frac{y}{2}$, then prove that

$$y \quad i\log\tan \frac{ix}{2} \quad \frac{}{4}$$

49. State and prove Gregory's series.

50. Prove that

$$\log\cos \quad \log 2 \quad \cos 2 \quad \frac{1}{2}\cos 4 \quad \frac{1}{3}\cos 6 \quad \cdots$$

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