MATHEMATICS

(5th Semester)

Course No. : MTMSEC–501T

(Integral Calculus)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

SECTION—A

Answer any *fifteen* questions : 1×15=15

1. Write down the value of

$$\int \frac{f'(x)}{f(x)}\, dx$$

2. Write down the value of

$$\int \frac{dx}{x^2 - a^2}$$

(2)

3. If

$$\frac{d}{dx}[f(x)] = F(x)$$

then find the value of $\int F(x)\, dx$.

4. Find the value of

$$\frac{d}{dx}\left[\int f(x)\, dx\right]$$

5. Write down the value of $\int a^x\, dx$.

6. Write down the value of

$$\int \frac{dx}{\sqrt{a^2 - x^2}}$$

7. If $f(x)$ is even, then what is the value of $\int_{-a}^{a} f(x)\, dx$?

8. Write down the value of

$$\int_{-\pi/2}^{\pi/2} \sin^3 x\, dx$$

9. Write True or False :

$$\int_0^{\pi/2} \log(\sin x)\, dx = \int_0^{\pi/2} \log(\cos x)\, dx$$

(3)

10. Find

$$\int_1^0 |x|\,dx$$

11. Find the value of

$$\int_a^a x\,(x^2)\,dx$$

12. Express $\int_a^b f(x)\,dx$ as limit sum.

13. Write down the reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$, when n is odd.

14. Write down the reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$, when n is even.

15. Write True or False :

$$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx$$

16. Find the value of $\int_0^{\pi/2} \sin^4 x\,dx$ by using reduction formula.

17. If $(n) = \int_0^{\pi/4} \tan^n x\,dx$, what is the value of $(n) + (n - 2)$?

(4)

18. Write down the reduction formula for $\sec^n x\,dx$.

19. Write down the geometrical interpretation of $\int_a^b f(x)\,dx$.

20. Write down the parametric equation of circle $x^2 + y^2 = r^2$.

21. Write down the parametric equation of astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

22. Write down the formula of length in Cartesian form.

23. Write down the formula of length in parametric form.

24. What is the length of circumference of a circle of radius a?

25. What is the surface area of the sphere of radius a?

26. What is the volume of the solid generated by the curve $y = f(x)$, intercepted between $x = a$ and $x = b$ and the axis of revolution about x-axis?

(5)

27. What is the surface area of the solid generated by the curve $x = f(y)$, intercepted between $y = a$ and $y = b$ and the axis of revolution about y-axis?

28. What is the volume of the sphere generated by the rotation of the circle $x^2 + y^2 = 4a^2$?

29. What is the volume of the solid generated by the revolution of the area bounded by the curve $r = f(\theta)$ and radii vectors $\theta = \theta_1$, θ_2 and revolution about initial line $\theta = 0$?

30. What is the volume of a paraboloid of revolution formed by revolving the parabola $y^2 = 4ax$ about the x-axis and bounded by the section $x = h$?

SECTION—B

Answer any *five* questions : 2×5=10

31. Evaluate :
$$\int \frac{\cos \sqrt{x}}{\sqrt{x}}\, dx$$

32. Evaluate :
$$\int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}}\, dx$$

(6)

33. Prove that
$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

34. Prove that
$$\int_0^{\pi/2} \log(\tan\theta)\, d\theta = 0$$

35. If $I_n = \int_0^{\pi/2} \sin^n x\, dx,\ n \geq 1$, then prove that
$$I_n = \frac{n-1}{n} I_{n-2}$$

36. If $I_n = \int \tan^n x\, dx,\ n \geq 1$, then prove that
$$I_n = \frac{\tan^n x}{n-1} - I_{n-2}$$

37. Find the length of the curve $x = e^\theta \sin\theta$, $y = e^\theta \cos\theta$; $\theta = 0$ and $\theta = \pi/2$.

38. Find the length of the curve $r = \theta^2$; $\theta = 0$ and $\theta = \sqrt{5}$.

39. Find the volume generated by revolving about x-axis, the area bounded by $y = \cos x$ between $x = 0$ and $x = \pi/2$.

40. Find the surface area generated by revolving about y-axis, the area bounded by $y = x^2$ between $y = 0$ and $y = \sqrt{2}$.

(7)

Answer any *five* questions :   5×5=25

41. Evaluate :
$$\int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$$

42. Evaluate :
$$\int \frac{dx}{x^4 + a^2}$$

43. Prove that
$$\int_0^{\pi/2} \log (\sin x)\, dx = -\frac{\pi}{2} \log \frac{1}{2}$$

44. Evaluate :
$$\lim_{n \to \infty} \left[\left(1 + \frac{1^2}{n^2}\right)\left(1 + \frac{2^2}{n^2}\right) \cdots \left(1 + \frac{n^2}{n^2}\right) \right]^{1/n}$$

45. If $u_n = \int_0^{\pi/2} x^n \sin x\, dx$, $n > 1$, then prove that
$$u_n + n(n-1)u_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$

46. If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x\, dx$; m, n being positive integers greater than 1, then prove that
$$I_{m,n} = \frac{n-1}{m+n} I_{m,n-2}$$

(8)

47. Find the total length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

48. Find the length of an arc of the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$.

49. Find the area of the surface generated by the arc of the parabola $y^2 = 4ax$ bounded by its latus rectum about x-axis.

50. Find the volume of the ellipsoid by the revolution of the ellipse
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
about the major axis.

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