2020/TDC/ODD/SEM/MTMP–501/265

TDC Odd Semester Exam., 2020
held in July, 2021

MATHEMATICS

(Pass)

(5th Semester)

Course No. : MTMP–501

(Dynamics and Statics)

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

GROUP—A

(Dynamics)

(*Marks* : 30)

UNIT—I

1. *(a)* If the relation between x and t for the motion of a particle is of the form $t \quad ax^2 \quad bx$, find the velocity v as a function of x and prove that the retardation of the particle is $2av^3$. 4

(b) Obtain expressions for tangential and normal components of acceleration of a particle moving in a plane curve. 6

2. *(a)* A particle is moving with SHM and while making an excursion from one position of rest to the other, the distances from the middle point of its path at three consecutive seconds are observed to be x_1, x_2, x_3. Prove that the time of a complete oscillation is

$$\frac{2}{\cos^{1}\frac{x_1 \quad x_3}{2x_2}}$$
5

(b) A body moving in a straight line *OAB* with SHM has zero velocity at the points *A* and *B* whose distances from *O* are *a* and *b* respectively and has a velocity *V* when half way between them. Show that the complete period is

$$\frac{(b \quad a)}{V}$$
5

UNIT—II

3. *(a)* Show that the path of a projectile in vacuum is a parabola. 6

(b) A particle is projected with an initial velocity u. If the greatest height attained by the particle be H, prove that the range R on the horizontal plane through the point of projection is

$$R \quad 4\sqrt{H \frac{u^2}{2g} \quad H}$$

4

4. *(a)* A particle aimed at a mark, which is in the horizontal plane through the point of projection, falls a feet short of it when the elevation is   and goes b feet too far when the elevation is . Show that if the velocity of projection in each case be same, then the proper elevation is

$$\frac{1}{2}\sin^{1}\frac{a\sin 2 \qquad b\sin 2}{a \quad b}$$

5

(b) A particle slides down the outside of a smooth vertical circle starting from rest at the highest point. Discuss the motion. 5

UNIT—III

5. *(a)* State the laws of direct impact. 3

(b) State the principle of conservation of energy. 2

(c) A ball impinges directly upon another ball at rest and is itself reduced to rest by impact. If half of the initial KE is destroyed in the collision, then find the coefficient of restitution. 5

6. *(a)* State and prove principle of conservation of linear momentum. 1+4=5

(b) A gun of mass M fires a shell of mass m horizontally and the energy of the explosion is such as would be sufficient to project the shell vertically to a height h. Show that the velocity of recoil is

$$\frac{2m^2gh}{M(M \quad m)}^{1/2}$$

5

GROUP—B

(Statics)

(*Marks* : 20)

UNIT—IV

7. *(a)* If three forces acting on a rigid body keep it in equilibrium, then prove that either all the forces meet at a point or all of them are parallel to one another. 5

(b) A uniform ladder resting in limiting equilibrium with one end on a rough horizontal floor and the other end against a smooth vertical wall, is inclined at an angle to the vertical. If be the coefficient of friction, show that tan 2 . 5

8. *(a)* Forces $\vec{P}, \vec{Q}, \vec{R}, \vec{S}$ act along the sides *AB*, *BC*, *CD*, *DA* of the cyclic quadrilateral *ABCD* taken in order where *A* and *B* are the extremities of the diameter. If they are in equilibrium, prove that

$$R^2 \quad P^2 \quad Q^2 \quad S^2 \quad 2\frac{PQS}{R}$$ 5

(b) Define *(i)* angle of friction and *(ii)* cone of friction. Also state the laws of limiting friction. 1+1+3=5

UNIT—V

9. *(a)* Define the centre of gravity of a body and show that it is unique. 2+3=5

(b) Find the position of the centre of gravity of a triangle formed by three uniform thin rods of the same material. 5

10. *(a)* Find the CG of an arc of a uniform circle of radius a subtending an angle 2 at the centre. 5

(b) Find the centre of gravity of the quadrant of the ellipse

$$\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$$

(The density of the plane lamina is considered to be uniform.) 5

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